MODAL AND COUPLING CHARACTERISTICS OF INHOMOGENEOUS DIELECTRIC SLAB WAVEGUIDES

E.F. Kuester, D.C. Chang and S.W. Maley

Department of Electrical Engineering
University of Colorado
Boulder, Colorado 80302

Abstract

A numerical scheme based on invariant imbedding methods is applied to the problem of calculating the propagation constants for the surface-wave modes of an inhomogeneous lossless dielectric slab. The method results in a first-order Riccati equation for the transverse wave impedance (or admittance), which is numerically integrated across the slab to yield the transverse resonance condition for each specific mode. The method is generalized to treat lossy structures, as well as coupling between slabs.

Introduction

Recently, some interest has been given to surface wave modes guided by various inhomogeneous dielectric structures for use in optical communication and processing systems. In the case of a single dielectric waveguide, analytical solution appears possible only for a few specific permittivity profiles[1,2], for the case of the near-grazing mode[3], and for the high-order modes in a multi-mode structure.[4] In general, however, the problem can only be solved numerically; most often the inhomogeneous profile is approximated by a finite number of equally-spaced homogeneous layers, and the modal characteristics are determined from a system of simultaneous equations obtained from the wave solutions within the various layers, and the boundary conditions at the interfaces.[5,6] Such an approach becomes cumbersome and time-consuming as the number of layers increases, and there is no satisfactory criterion for determining the layer size required to obtain a specified accuracy. In this paper, a different method, utilizing the transverse impedance concept, is developed to analyze the modal and coupling characteristics in single or multiple slabs with arbitrary permittivity profiles. A first-order Ricatti differential equation for the impedance (or admittance) is formulated and numerically integrated across the slab(s) in order to satisfy the transverse resonance condition for any specific mode. It appears that this approach not only is rapidly convergent and highly accurate, but also can provide useful insight into the design of dielectric waveguides.

Formulation

We assume that the guide consists of a single inhomogeneous lossless slab between $x = 0$ and $x = d$, with permittivity profile $\varepsilon(x) = \varepsilon_o \varepsilon_r(x)$. The slab is bounded on both sides by infinite homogeneous lossless dielectrics: for $x>d$ of permittivity $\varepsilon_1 = \varepsilon_o \varepsilon_{r_1}$ and for $x<0$ of permittivity $\varepsilon_2 = \varepsilon_o \varepsilon_{r_2} \geq \varepsilon_1$, with no loss of generality.

The system is infinite and uniform in the y-direction, and propagation in the z-direction is assumed, so that field dependence is $\exp(j\omega t - jk_o \alpha z)$. The fields can be shown from Maxwell's equations to satisfy

$$(K(u)f'(u))' + \gamma^2(u;\alpha)f(u) = 0 \qquad (1)$$

where f is E_y or H_y, K is 1 or $1/\varepsilon_r$, and γ^2 is $(\varepsilon_r - \alpha^2)$ or $(\varepsilon_r - \alpha^2)/\varepsilon_r$ for TE or TM modes respectively. In addition, at the boundaries we require

$$f'(u_i) = \delta_i \Gamma_i f(u_i) \quad i = 1,2 \qquad (2)$$

where $u_1 = k_o d$, $u_2 = 0$, $\Gamma_i = (\alpha^2 - \varepsilon_{r_i})^{\frac{1}{2}}$, and δ_i is -1, $+1$, $-\varepsilon_r(u_1)/\varepsilon_{r_1}$ or $\varepsilon_r(0)/\varepsilon_{r_2}$ for TE modes ($i = 1,2$) or TM modes ($i = 1,2$) respectively. We have set $u = k_o x$ and $\alpha = \beta/k_o$, where $k_o^2 = \omega^2 \mu_o \varepsilon_o$. We require that Γ_1 and Γ_2 be real and positive in order to have an evanescent field outside the slab.

We now make use of the Prüfer transformation[7,8]

$$f(u) = r(u)\sin\theta(u)$$
$$K(u)f'(u) = r(u)\cos\theta(u) \qquad (3)$$

so that

$$\theta'(u) = K(u)^{-1}\cos^2\theta(u) + \gamma^2(u;\alpha)\sin^2\theta(u) \qquad (4)$$

with boundary conditions

$$\theta(0) = \operatorname{arccot}(\delta_2' \Gamma_2) \qquad (5)$$

$$\theta(k_o d) = \operatorname{arccot}(\delta_1' \Gamma_1) + (p-1)\pi \qquad (6)$$

where δ_1' is -1 or $-1/\varepsilon_{r_1}$ and δ_2' is $+1$ or $+1/\varepsilon_{r_2}$ for TE or TM modes, respectively, and p (as yet) is any integer. Thus the propagation constant α_p corresponding to the p^{th} surface mode, if it exists, will satisfy

$$P(\alpha_p) \equiv \theta(k_o d;\alpha_p) - \operatorname{arccot}(\delta_1' \Gamma_1) - (p-1)\pi = 0 \qquad (7)$$

where $\theta(u;\alpha)$ denotes the solution of (4) subject to (5). Using a derivation similar to that of [7] or [8], it is not difficult to show that, since $\alpha^2 > \alpha^2_{min} = \varepsilon_{r_2}$, and $\chi(u;\alpha) \equiv \partial/\partial\alpha(\theta(u;\alpha)) < 0$ for all u and α under consideration, α_p is bounded by $\alpha^2_{min} < \alpha_p^2 < \alpha^2_{max} = \varepsilon_{r_{max}}$ for any surface wave mode, so that the number of such modes is given by

$$p_{max} = \text{greatest integer } \{[\theta(k_o d;\alpha_{min}) - \operatorname{arccot}(\delta_1'(\varepsilon_{r_2} - \varepsilon_{r_1})^{\frac{1}{2}})]/\pi + 1\}$$

and $1 \leq p \leq p_{max}$.

Now if we let

$$Z(u) = \cot\theta(u) (= j\omega\varepsilon_o E_z/H_y),$$

and

$$Y(u) = \tan\theta(u)$$

then Z and Y are, respectively, a normalized wave impedance and admittance in the case of TM-modes, and from (4) we have

$$Z'(u) = -(Z^2(u)/K(u) + \gamma^2(u;\alpha)) \qquad (8)$$

with a similar equation for Y. This equation provides a numerically more efficient scheme than (4) because of the absence of trigonometric evaluations; however, for the p^{th} order mode there are (p-1) poles in Z and p poles in Y as a result of field oscillations within the slab. This is avoided numerically by switching between Z and Y during integration across the slab whenever the absolute value of one becomes larger than some specified value, say unity.[9] In addition the order of the mode can be determined by counting the number of poles in Z, say m, so that

$$\theta(k_o d) = m\pi + \text{arccot}\ Z(u_o)$$

To integrate Y or Z across the slab accurately, a Runge-Kutta method with assigned error bounds on single steps is employed[10], and, combined with adaptive stepping procedures, an overall error bound is maintained on the integration.

To find the roots of (7), Newton's method can be used, with successive corrections of the form

$$\alpha_{n+1} = \alpha_n - P(\alpha_n)/P'(\alpha_n)$$

so that $\chi(u_o;\alpha)$ is required. But χ satisfies

$$\chi' = \{[\gamma^2 - K^{-1}]\cdot 2Z\chi - 2\alpha K\}/(1+Z^2)$$

and a similar relation involving Y, where we have made use of the fact that $\frac{\partial}{\partial\alpha}\gamma^2 = -2\alpha K$ for both the TE and TM case. Hence χ can be integrated across the slab simultaneously with Y and Z.

Lossy and Multiple Slabs

If any part of the guide is lossy, so that ε_r is complex, but $Im(\varepsilon_r) << Re(\varepsilon_r)$ in a practical situation, this method can be extended merely by making the appropriate quantities complex, choosing the proper branch of the square root, and using the corresponding α for the lossless slab as an initial guess for Newton's method.

For multiple slabs which are separated by regions of constant permittivity, wide compared to one of the slabs, it can be seen that in the intermediate regions (11) has the solution (for TM modes)

$$Z(u;\alpha) = \frac{\gamma}{\varepsilon_r}\,\frac{\varepsilon_r Z(u_1;\alpha)\cos\gamma(u-u_1) - \gamma\sin\gamma(u-u_1)}{\varepsilon_r Z(u_1;\alpha)\sin\gamma(u-u_1) + \gamma\cos\gamma(u-u_1)}$$

i.e., the transmission line impedance relation, and so the tedious numerical integration along the intermediate region is avoided.

Results and Discussion

This method has been applied to the guide obtained by diffusing a layer of dielectric material onto a substrate of lower dielectric constant with an air interface above the diffused layer. In Fig. 1 the dispersion curve was obtained for several diffusion widths, and in Fig. 2 for two other values of substrate permittivity. In Fig. 3 a symmetrical guide embedded in the substrate is shown. For all these guides, the diffusion profile was approximated by a sinusoid for convenience. In all results, $\varepsilon_{r_{max}} = 1.53$ and $\Delta\beta/k_o = \alpha - \alpha_{min}$.

It should be noted that, the guides of Figs. 1 and 2, unlike symmetric guides, exhibit non-zero cutoff frequences for all modes. The results seem to indicate that less distortion of signals due to dispersion (e.g. in pulse propagation) can be achieved at a given frequency by (a) making $(\varepsilon_{r_{max}} - \varepsilon_{r_2})$ as small as practical, (b) adjusting the diffusion width d, for as gentle a transition as possible, and/or (c) increasing slab width (while remaining in single-mode operation). In addition, operation at relatively low frequences is better in the symmetric guides of Fig. 3, since at low frequencies the guides of Figs. 1 and 2, when not cut off, are highly dispersive (since on these normalized dispersion curves, the horizontal line has no dispersion). The curves were obtained on a CDC 6400 computer (to 5-digit accuracy) in an average time of 45 seconds for a set of curves with an average of 30 data points.

References

1. Kirchhoff, H., IEE Conf. Publ. 71, p.69, 1970.
2. Marcatili, E.A.J., IEE Conf. Publ.71, p. 83, 1970.
3. Brekhovskikh, L.M., Waves in layered media, Academic Press, N.Y., 1960.
4. Kurtz, C.N., and W. Streifer, IEEE Trans. MTT, vol. 17, p. 11-15, 1969.
5. Clarricoats, P.J.B., and K.B. Chan, Electronics Letters, vol. 6, no. 22, p. 694-695, 1970.
6. Graham, W.R.,and W.J.Karzas, EMP Theoretical Note 67, A.F. Weapons Lab., October 1971.
7. Godart, M., Math. of Comp. 20,95, p. 399-406, 1966.
8. Birkhoff, G., and G.-C. Rota, Ordinary Differential Equations, Ginn,Boston, 1962, pp. 257-262.
9. Scott, M.R., et al., Computing 4, p.10-23, 1969.
10. Fox, L., Numerical Solution of Ordinary and Partial Differential Equations, Addison-Wesley, Reading, 1962,p. 24.

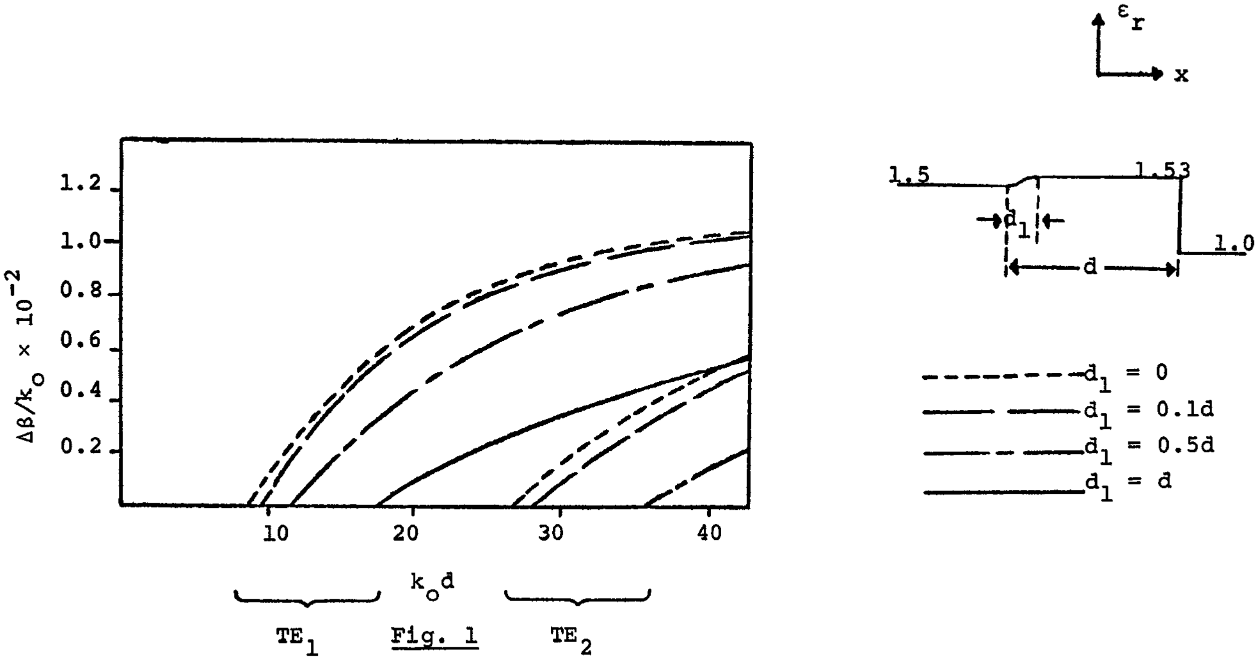

Fig. 1

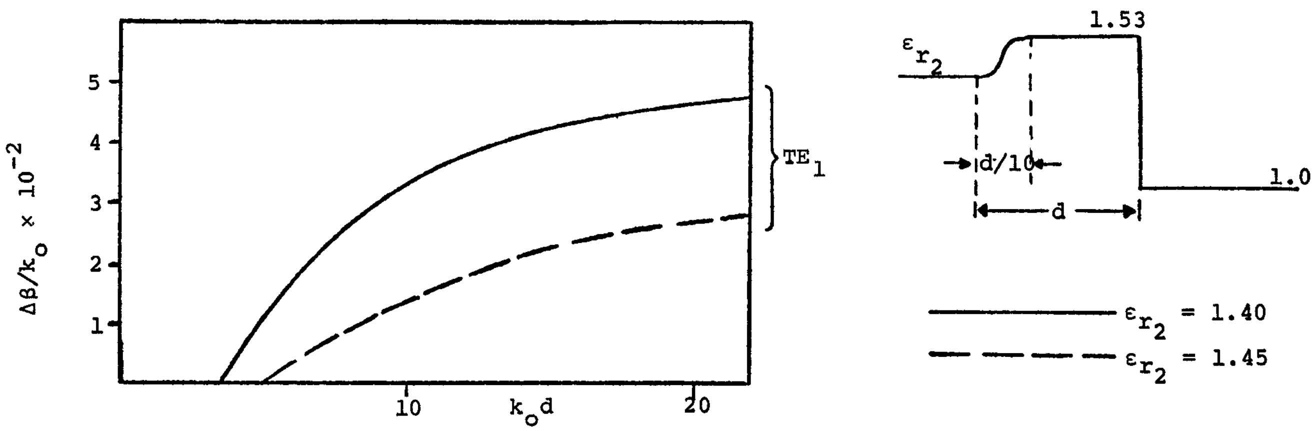

Fig. 2

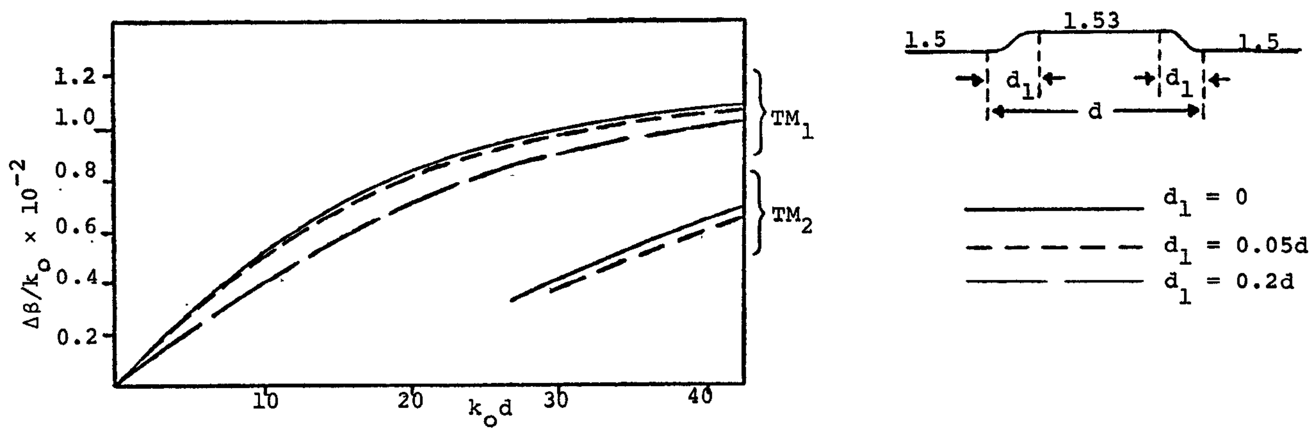

Fig. 3